Ant Colony Optimization for Model Predictive Control for Blood Glucose Regulation

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ABSTRACT

This paper presents an adaptation of the Ant System method to find the optimal control input for blood glucose regulation using Model Predictive Control (MPC). The Ant System optimization method was implemented to solve a linear MPC problem and performance was compared with the interior point method for optimization. The Ant System was found to perform well for the linear MPC problem and has the advantage over the interior point method as it can extended for use with non-linear MPC problems.

Categories and Subject Descriptors

I.2.8 [**Artificial Intelligence**]: Problem Solving, Control Methods, and Search—*Control theory*

General Terms

Algorithms, Design

Keywords

Artificial pancreas, ant colony optimization, model predictive control

1. INTRODUCTION

Diabetes is a disease that affects millions of people all over the world. In a bid to improve diabetics' quality of life, much work is going into the development of the artificial pancreas and other methods to deliver insulin in an automated method that does not require patients to carry insulin on them nor worry about the administration of dosage. Model Predictive Control (MPC) has been found to be a good method to compute the insulin infusion to regulate blood glucose [3]. Much research has also been done on finding models that simulate the human glucose-insulin interaction. However, for implementation in a embedded system, the model cannot be overly complicated, yet needs to simulate the human system accurately. The minimal model proposed in [1] has only three differential equations and the model was then linearized. For a linear system, finding the optimal control input (the insulin infusion rate) for the model predictive control is a convex optimization problem.

However a linear model has its limitations and using MPC with a non-linear model like those proposed in [4] and [10], the optimization to find the control input may be a non-convex problem. Popular methods to solve convex optimization, like the interior point method and conjugate gradient methods, may not work so well on non-linear problems. Stochastic based techniques using biologically inspired computing methods such as Ant Colony Optimization [6] and other Swarm Intelligence methods, Artificial Immune Systems [5] and Genetic Algorithm [2], are able to find a minimum to non-convex optimization problems as the probabilistic nature prevents the search being trapped in local minima. This paper adapts the Ant System method [6] to find optimal control inputs for the Model Predictive Control.

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2. MATERIALS AND METHODS

2.1 Minimal Model of Glucose-Insulin Dynamics

The Minimal Model was proposed by Bergman et al [1] and contains a minimum number of parameters to be estimated. The glucose-insulin interaction is described by three differential equations. Modifying the minimal model to include a meal disturbance and the insulin infusion as proposed by Fisher [7], the model used is as follows

$$\dot{G}(t) = -p_1 G(t) - X(t)[G(t) + G_b] + D(t) \tag{1a}$$

$$\dot{X}(t) = -p_2 X(t) + p_3 I(t) \tag{1b}$$

$$\dot{I}(t) = p_4 (G(t) - h)^+ - n[I(t) + I_b] + u(t)/V \tag{1c}$$

where $G(t)$ is the difference between plasma glucose concentration (mmol/L) and the basal, $I(t)$ is the difference between plasma insulin concentrations (mU/L) and the basal value; and $X(t)(\text{min}^{-1})$ is proportional to the insulin concentration in the plasma and mimics the effective insulin activity. p_1 is the rate of plasma glucose decay, p_2 the disappearance rate of insulin in the remote compartment, p_3

is the rate insulin enters the remote compartment, p_4 is the rate of insulin secreted from the pancreas entering blood plasma and h is the pancreatic insulin secretion threshold level.

$D(t)$ (mmol/L) is the glucose increase due to the meal disturbance and the model used in this paper is the one proposed by Fisher [7], modeled by the decaying exponential function $D(t) = 1.157exp(-0.05t)$.

2.2 Model Predictive Control

In Model Predictive Control (MPC) the N_p future states of the system are predicted by a known model of the dynamic process of the system. The predicted states are used to find the optimal control sequence of control horizon N_c.

The three differential equations of the minimal model were linearized [8] about the steady state of $G(0)=X(0)=I(0)=0$ and were converted into discrete time state-space of the form

$$x_m(k+1) = A_m x_m(k) + B_m u(k) + D_m d(k) \tag{2a}$$
$$y(k) = C_m x_m(k) \tag{2b}$$

By defining the augmented state as $x(k) = [\Delta x_m(k)^T y(k)]^T$ the state space model obtained is

$$x(k+1) = Ax(k) + Bu(k) + Dd(k) \tag{3a}$$
$$y(k) = Cx(k) \tag{3b}$$

where

$$A = \begin{bmatrix} A_m & O \\ C_m A_m & I \end{bmatrix}, B = \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix}, C = \begin{bmatrix} O & I \end{bmatrix}, D = \begin{bmatrix} D_m \\ O \end{bmatrix}$$

By defining [9]:

$$U = \begin{bmatrix} u(k)^T & u(k+1)^T & \cdots & u(k+N_c-1)^T \end{bmatrix}^T$$

$$Y = \begin{bmatrix} y(k+1|k)^T & y(k+2|k)^T & \cdots & y(k+N_p|k)^T \end{bmatrix}$$

where Y is the vector which contains the N_p predicted future values based on the current value of $y(k)$ and the N_c future control inputs U.

$$Y = Fx(k) + \Phi U \tag{4}$$

where

$$F = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^{N_p} \end{bmatrix}^T$$

$$\Phi = \begin{bmatrix} CB & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N_p-1}B & CA^{N_p-2}B & \cdots & CA^{N_p-N_c}B \end{bmatrix}$$

The control sequence U was then computed by solving the minimization problem where the cost function is given by

$$J = (R_s - Y)^T (R_s - Y) + U^T \bar{R} U \tag{5}$$

In practice, as only the plasma glucose concentration, G, is measurable, the state is estimated via an observer

$$\hat{x}_m(k+1) = A_m \hat{x}_m(k) + B_m u(k) + K_{ob}(y(k) - C_m \hat{x}_m(k))$$

where K_{ob} is the observer gain. The estimated state is used in place of the state in equations (4) and (5).

2.3 Ant System

Ant System is an optimization meta-heuristic developed by Dorigo et al [6] which is inspired by the way groups of ants are able to find the shortest route from the food source to the ant colony. It was first presented as a method solve combinatorial optimization problems such as the Travelling Salesman Problem and can be described as follows.

At each cycle, each of the m artificial ants traverse a path travelling through n cities. The distance between city i and city j is d_{ij}. The ant is constrained such that it cannot visit a city more than once during a cycle. At the end of the cycle, the total distance travelled by each ant, L_k, is calculated.

The transition probability between cities i and j for k-th ant is

$$p_{ij}^k(t) = \frac{[\tau_{ij}(t)]^\alpha . [1/d_{ij}]^\beta}{\sum [\tau_{ik}(t)]^\alpha . [1/d_{ik}]^\beta} \tag{6}$$

where τ_{ij} is the intensity of the pheromone trail left on the path between i and j. α and β are tuning parameters to control the relative importance of pheromone trail versus distance.

After all m ants have completed the tour, the trail intensity is updated by

$$\tau_{ij}(t+n) = \rho . \tau_{ij}(t) + \Delta\tau_{ij} \tag{7}$$

where ρ is related to the evaporation rate of the trail.

The trail deposited between i, j is

$$\Delta\tau_{ij} = \sum_{k=1}^{m} \Delta\tau_{ij}^k \tag{8}$$

where

$$\Delta\tau_{ij}^k \propto 1/L_k$$

2.4 Ant System for Finding an Optimal Control Sequence for MPC

This paper adapts the Ant System optimization [6] to find the optimal sequence of control inputs U, a vector of length N_c for MPC. The aim is to find the U that minimizes the cost function, J, given by equation (5).

The discrete nature of Ant System optimization can also be applied to the insulin dosage computation. The insulin infusion rates are not required to take on continuous values and can be set to take integer values. The need to constrain the insulin dosage due to the physical limits of hardware and the patient is automatically done by appropriately selecting the values each element of U, $U(t)$, can take.

The constraint by the Travelling Salesman Problem that each city, or node, be only visited once in a tour is no longer applicable to this problem. The values that $U(t)$ take does not need to be unique and the same value can be repeated in the entire control sequence.

First, the m ants were placed on the n nodes which correspond to the values the insulin infusion rate, $U(t)$, can take.

Then, the next node was selected, based on p_{jt} , the probability that node j is chosen at time instant t. This is repeated for all $U(t), 1 \leq t \leq N_c$. For each of the m ants, the predicted sequence of outputs Y based on current output y and control sequence U was obtained and the cost function J was calculated.

Then, the trail values and probabilities were updated as follows

For the k-th ant,

$$\Delta\tau_k = \frac{J_{max} - J_k}{J_{max} - J_{min}} \tag{9}$$

where J_k is the cost function for the k-th ant, J_{max} and J_{min} are the maximum and minimum values of the cost function J obtained by all the m ants in the cycle.

$$\Delta\tau_{jt} = \sum\nolimits_{k \in allowed} \Delta\tau_k \tag{10}$$

$k \in allowed$ if the k-th ant was at node j at time instant t.

More trail is deposited if the value of cost function J is lower when a node j is used at time instant t.

The trail is then updated by

$$\tau_{jt} = \rho\tau_{jt} + \Delta\tau_{jt} \tag{11}$$

Correspondingly, the probability of node j chosen at time instant t was updated before the next cycle using

$$p_{jt} = \frac{\tau_{jt}}{\sum_{j=1}^{n} \tau_{jt}} \tag{12}$$

The process is repeated until the maximum number of allowable cycles is reached or when a stagnation point is reached such that the difference of J between cycles is below a certain threshold.

3. RESULTS AND DISCUSSION

The adapted Ant System was tested and compared to the interior point method using various prediction horizons, N_p, and control horizons, N_c.

The number of nodes n=160 and the values of U at any time index t are constrained to integer values between 0 to 159 mU/min. The number of ants used was 480, 3 times the number of nodes. A value of 0.5 was used for ρ which is related to the rate of evaporation of the pheromone trail. The number of iteration cycles was capped to 30 and the stagnation tolerance was set to 1.

The patient specific parameters for the minimal model is obtained from [11]. The parameters used are $p_1 = 1.109 \times 10^{-2}$, $p_2 = 6.1259 \times 10^{-1}$, $p_3 = 7.6286 \times 10^{-6}$, $p_4 = 1.9424 \times 10^{-2}$, $V_1 = 12$ and $n = 5.6 \times 10^{-3}$. The basal plasma glucose G_b and insulin I_b concentrations used were 7.4233 mmol/L and 19.033 mU/L respectively. The initial plasma glucose concentration, G_0, was 18 mmol/L and the initial plasma insulin concentration I_0 used was the basal value, I_b. There was a meal disturbance at $t = 250$ min.

The reference setpoint used was the basal plasma glucose level G_b. The goal is to bring down the plasma glucose concentration to the setpoint, however hypoglycemia must be avoided and G should not go below the lower bounds. It is also desirable for G to reach the reference set point at steady state.

The prediction horizons, N_p, tested were $10, 20, 50$ and 100. The control horizons, N_c, tested were $2, 4, 10, 20, 50$, noting that $N_c \leq N_p$. As the states are not all known, the state estimator with observer poles at 0.01, 0.5 and 0.8 was used. The state space model was discretized for a sampling time of 5 minutes and the simulation run for 1000 minutes.

Using the interior point method, higher values of N_c resulted in the increased total amount of insulin delivered and led to G reaching close to hypoglycemic level. Increasing N_c had no noticeable effect on the control input U for $N_p = 10$ and 20. For the interior point method, using the lower N_p of 10 and 20 produced non-zero values of the insulin infusion rate U even after the set point has been reached.

Using the adapted ant system method, increasing the control horizon N_c led to an overall increase in the amount of insulin delivered and also larger random spikes of of U, leading to a lower G at steady state and in some cases reaching the lower bound into hypoglycemia. At low values of N_c (2,4,5,10), increasing N_p improves the control performance, however this is not the case at higher values of N_c.

As the performance varies with different combinations of N_p and N_c, a comparison of the adapted ant system and the interior point methods would depend also on the values of N_p and N_c used.

For $N_p = 20$, the adapted ant system method (Figure 1) resulted in a desirable regulation of G and performed much better than the interior point method (Figure 2). The interior point method resulted in G reaching almost at hypoglycemic levels.

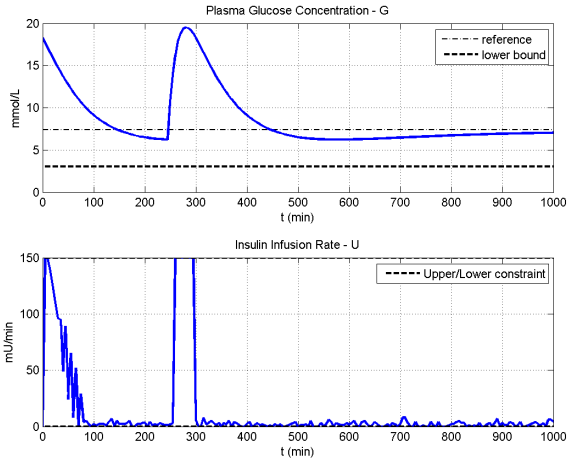

Figure 1: Ant System method with $N_p = 20$, $N_c = 4$

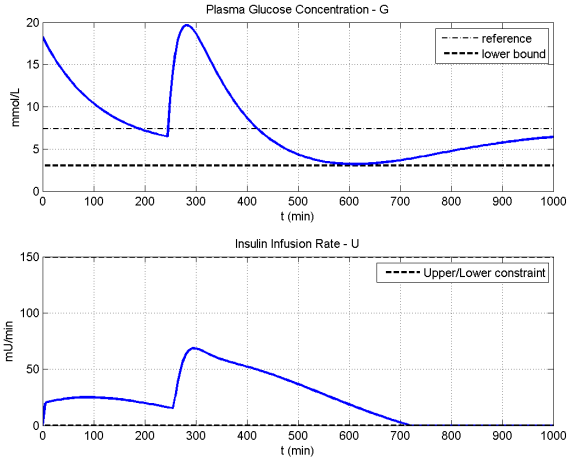

Figure 2: Interior Point method with $N_p = 20$, $N_c = 4$

For $N_p = 50$ the adapted ant system method performed adequately well without any hypoglycemia (Figure 3) although it was slightly offset from the set point even at $t = 1000$ which the interior point method achieved (Figure 4).

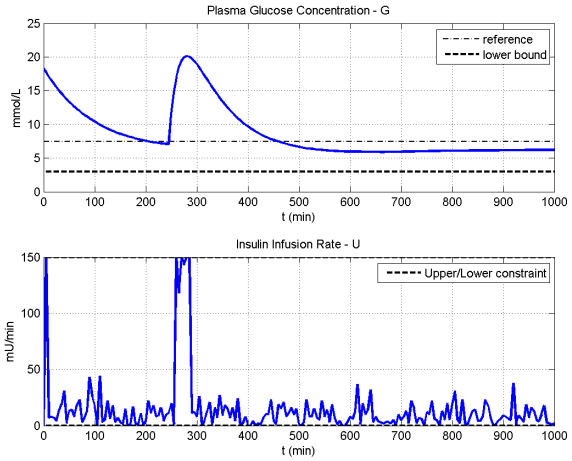

Figure 3: Ant System method with $N_p = 50$, $N_c = 10$

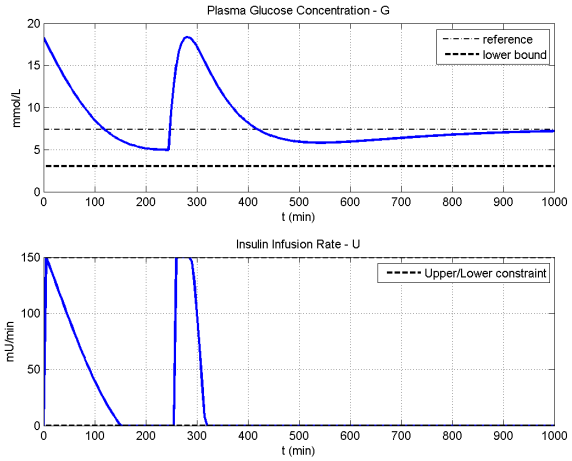

Figure 4: Interior Point method with $N_p = 50$, $N_c = 10$

Beside variation of performance with the different prediction and control horizons, N_p and N_c, the performance of both methods can be varied by changing the relative weightage of the components of the cost function J.

In all test runs, the adapted Ant Systems method took longer than the Interior Point method to compute U. However, the time was well within the sampling time of 5 minutes in an artificial pancreas.

4. CONCLUSION

The Ant System originally proposed to solve a combinatorial optimization problem was adapted for a convex optimization problem to find the optimal control input for MPC. The Ant System method performed well and can be improved further by judicious choice of MPC parameters and weightage matrices of the cost function. As the Ant System method of optimization is stochastic in nature, it is feasible to extend its application to non-linear MPC and gain the advantages of a more accurate non-linear model of the glucose-insulin dynamics in humans.

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